

## 2.3 Answer Key

### Practice 2.3-1-1:

Identify the degree, leading term, and leading coefficient of the polynomial  $4x^2 - x^6 + 2x - 6$ .

The highest power of  $x$  is 6, so the degree is 6. The leading term is the term containing that degree,  $-x^6$ . The leading coefficient is the coefficient of that term,  $-1$ .

### Practice 2.3-2-1:

Using the given polynomial, find the degree, leading term, leading coefficient, multiplicity, maximum number of turning points, and real zeros.

$$f(x) = -4x(x - 1)(x + 3)^2$$

#### Degree:

To find degree, we just need to find the highest power of the  $x$  in the expanded form. We have

$$f(x) = -4x(x - 1)(x + 3)^2$$

$$f(x) = -4x(x - 1)(x^2 + 6x + 9)$$

$$f(x) = (-4x^2 + 4x)(x^2 + 6x + 9)$$

$$f(x) = -4x^4 - 24x^3 - 36x^2 + 4x^3 + 24x^2 + 36x$$

$$f(x) = -4x^{\textcircled{4}} - 20x^3 - 12x^2 + 36x$$



**The highest degree  
of the polynomial**

Degree is 4.

#### Leading Term:

Since the highest degree of  $x$  is 4, the term that contains  $x^4$  is the leading term.

Leading term:  $-4x^4$ .

#### Leading Coefficient:

Since the leading term is  $-4x^4$ , the coefficient of the leading term is the leading coefficient.

Leading coefficient: -4.

### Multiplicity:

If  $(x - r)^m$  is a factor of a polynomial  $f$  and  $(x - r)^{m+1}$  is not a factor of  $f$ , then  $r$  is called a real zero of multiplicity  $m$  of  $f$ .

Format:

$$f(x) = (x - r_1)^{m_1}(x - r_2)^{m_2}(x - r_3)^{m_3} \dots (x - r_{n-1})^{m_{n-1}}(x - r_n)^{m_n}$$

$m_1, m_2, m_3 \dots m_{n-1}, m_n$  are the multiplicities.

In this question we have

$$f(x) = -4x(x - 1)(x + 3)^2$$

We can rewrite as:

$$f(x) = -4(x - 0)^{\textcircled{1}}(x - 1)^{\textcircled{1}}(x + 3)^{\textcircled{2}}$$

Thus  $m_1 = 1, m_2 = 1, m_3 = 2$

### Turning Points:

The most turning points = The highest degree of the polynomial - 1

The most turning points =  $4 - 1 = 3$

### Real Zeros:

If  $f$  is a function and  $r$  is a real number for which  $f(r) = 0$ , then  $r$  is called a real zero of  $f$ .

Thus:  $r$  is a real zero of a polynomial function  $f = r$  is an  $x$ -intercept of the graph of  $f = r$  is a real solution to the equation  $f(x) = 0$

Format:

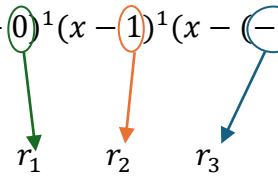
$$f(x) = (x - r_1)^{m_1}(x - r_2)^{m_2}(x - r_3)^{m_3} \dots (x - r_{n-1})^{m_{n-1}}(x - r_n)^{m_n}$$

$r_1, r_2, r_3 \dots r_{n-1}, r_n$  are real zeros of the polynomials.

In this question we have

$$f(x) = -4x(x - 1)^1(x + 3)^2$$

Rewrite as

$$f(x) = -4(x - \textcircled{0})^1(x - \textcircled{1})^1(x - \textcircled{-3})^2$$


$r_1$   $r_2$   $r_3$

Real zeros are 0, 1, -3