2.3 Answer Key

Practice 2.3-1-1:

Identify the degree, leading term, and leading coefficient of the polynomial $4x^2 - x^6 + 2x - 6$.

The highest power of x is 6, so the degree is 6. The leading term is the term containing that degree, $-x^6$. The leading coefficient is the coefficient of that term, −1.

Practice 2.3-2-1:

Using the given polynomial, find the degree, leading term, leading coefficient, multiplicity, maximum number of turning points, and real zeros.

$$f(x) = -4x(x-1)(x+3)^2$$

Degree:

To find degree, we just need to find the highest power of the x in the expanded form. We have

$$f(x) = -4x(x-1)(x+3)^2$$

$$f(x) = -4x(x-1)(x^2+6x+9)$$
$$f(x) = (-4x^2+4x)(x^2+6x+9)$$
$$f(x) = -4x^4 - 24x^3 - 36x^2 + 4x^3 + 24x^2 + 36x$$
$$f(x) = -4x^4 - 20x^3 - 12x^2 + 36x$$

The highest degree of the polynomial

Degree is 4.

Leading Term:

Since the highest degree of x is 4, the term that contains x^4 is the leading term.

Leading term: $-4x^4$.

Leading Coefficient:

Since the leading term is $-4x^4$, the coefficient of the leading term is the leading coefficient.

Leading coefficient: -4.

Multiplicity:

If $(x-r)^m$is a factor of a polynomial f and $(x-r)^{m+1}$ is not a factor of f, then r is called a real zero of multiplicity m of f.

Format:

$$f(x) = (x-r_1)^{m_1}(x-r_2)^{m_2}(x-r_3)^{m_3} \dots \dots (x-r_{n-1})^{m_{n-1}}(x-r_n)^{m_n}$$

$m_1, m_2, m_3 \dots \dots m_{n-1}, m_n$ are the multiplicities.

In this question we have

$$f(x) = -4x(x-1)(x+3)^2$$

We can rewrite as:

$$m_1 \quad m_2 \quad m_3$$

$$f(x) = -4(x-0)^{1}(x-1)^{1}(x+3)^{2}$$

Thus $m_1 = 1, m_2 = 1, m_2 = 2$

Turning Points:

The most turning points = The highest degree of the polynomial−1

The most turning points = 4– 1 = 3

Real Zeros:

If f is a function and r is a real number for which $f(r) = 0$, then r is called a real zero of f.

Thus: r is a real zero of a polynomial function f = r is an x-intercept of the graph of f = r is a real solution to the equation $f(x) = 0$

Format:

$$f(x) = (x-r_1)^{m_1}(x-r_2)^{m_2}(x-r_3)^{m_3} \dots \dots (x-r_{n-1})^{m_{n-1}}(x-r_n)^{m_n}$$

$r_1, r_2, r_3 \dots r_{n-1}, r_n$ are real zeros of the polynomials.

In this question we have

$$f(x) = -4x(x-1)^1(x+3)^2$$

Rewirte as

$$f(x) = -4(x-0)^1(x-1)^1(x-(-3))^2$$

$$r_1 \quad r_2 \quad r_3$$

Real zeros are 0, 1, -3