2.4 Answer Key

Practice 2.4-1-1:

Using the remainder theorem to evaluate a polynomial.

$f(x) = 2x^5 - 3x^4 + 2x^3 + 4x - 1$ at x= - 1
$f(-1) = 2(-1)^5 - 3(-1)^4 + 2(-1)^3 + 4(-1) - 1$
$f(-1) = 2(-1) - 3(1) - 2(-1) + 4(-1) - 1$
$f(-1) = -2 - 3 + 2 - 4 - 1$
$f(-1) = -8$

Practice 2.4-2-1:

List all possible rational zeros of

$$f(x) = 2x^3 + x^2 - 4x + 1$$

The Rational Zero Theorem tells us that if $\frac{p}{q}$ is a zero of $f(x)$, then p is a factor of 1 and q is a factor of 2.

$$\frac{p}{q} = \frac{factor\ of\ constant\ term}{factor\ of\ leading\ coefficient}$$

$$= \frac{factor\ of\ 1}{factor\ of\ 2}$$

The factors of 1 are ± 1 and the factors of 2 are ± 1 and ± 2. The possible values for $\frac{p}{q}$ are ± 1 and ± 12.

Practice 2.4-3-1:

Using the rational zero theorem to final rational zeros.

$$f(x) = 2x^3 + x^2 - 4x + 1$$

From the previous Practice 2.4-2-1, we have found potential zeros

$$x = \pm 1, \pm \frac{1}{2}$$

The factors of 1 are ±1 and the factors of 2 are ±1 and ±2. The possible values for p/q are ±1 and ±12. These are the possible rational zeros for the function. We can determine which of the possible zeros are actual zeros by substituting these values for x in $f(x)$.

$$f(-x) = 2(-1)^3 + (-1)^2 - 4(-1) + 1 = 4$$

$$f(1) = 2(1)^3 + (1)^2 - 4(1) + 1 = 0$$

$$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) + 1 = 3$$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1 = -\frac{1}{2}$$

Of those, -1, $\frac{-1}{2}$, and $\frac{1}{2}$ are not zeros of $f(x)$. 1 is the only rational zero of $f(x)$.

Practice 2.4-5-1:

Find zeros of a polynomial function

q : factors of 2 are ± 1, ± 2

p : factors of 6 are ± 1, ± 6, ± 2, ± 3

$$f(x) = 2x^3 - 3x^2 - 11x + 6$$

1. List possible rational zeros

$$\frac{p}{q} = \pm \frac{factors\ of\ constant}{factors\ of\ leading\ coefficient}$$

$$\frac{p}{q} = \pm\frac{1}{1}, \pm\frac{6}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{1}{2}, \pm\frac{6}{2}, \pm\frac{2}{2}, \pm\frac{3}{2}$$

$$\frac{p}{q} = \pm 1, \pm 6, \pm 2, \pm 3, \frac{1}{2}, \pm\frac{3}{2}$$

2. Test values using Remainder Theorem
   Plug into the polynomial.
   If f(x) = 0, then x is a real zero.

   Test values if x=1

$$f(1) = 2(1)^3 - 3(1)^2 - 11(1) + 6 = -6$$

   Test values if x=2

$$f(2) = 2(2)^3 - 3(2)^2 - 11(2) + 6 = -12$$

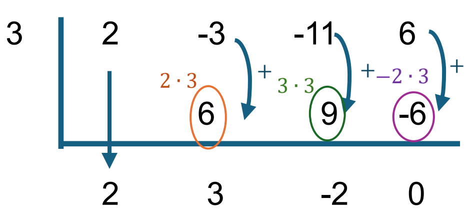
   Test values if x=3

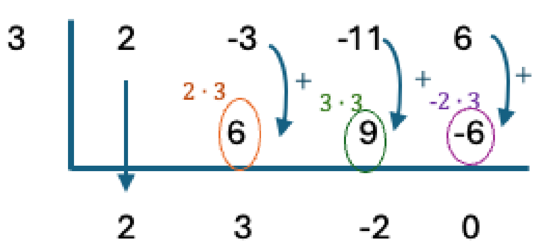
$$f(3) = 2(3)^3 - 3(3)^2 - 11(3) + 6 = 0$$

X=3

Once we find one real zero, we can stop at this step and start the next step.

3. Use Synthetic Division
   Divide the polynomial by **the value of x we found in step 2 (real zero).**
   This gives a simpler polynomial. Repeat the process.





4. Solve the remaining polynomial
   Factor or use the quadratic formula to find more zeros.

$$2x^2 + 3x - 2 = 0$$

$$(x-1)(x+4)=0$$

$$(x-\frac{1}{2})(x+\frac{4}{2})=0$$

$$(x-\frac{1}{2})(x+2)=0$$

$$x=\frac{1}{2}, x=-2$$

Overall real zeros are: $\boldsymbol{x=\frac{1}{2}, x=-2, x=3}$