2.5 Answer Key

Practice 2.5-1-1:

Find the domain of the rational function.

$$f(x) = \frac{x+2}{x^2 - x - 2}$$

1. Set the denominator ≠ 0

$$x^2 - x - 2 \neq 0$$

2. Solve for x
   Find the values that make the denominator equal to 0.

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2, x = -1$$

3. Exclude those x-values from the domain
   These are the restrictions.

$$x \neq 2, x \neq -1$$

4. Write the domain
   Use interval notation to show all allowed x-values.

$$(-\infty, -1) \cup (-1,2) \cup (2, \infty)$$

Practice 2.5-2-1:

Find the vertical asymptotes and removable discontinuities of the function.

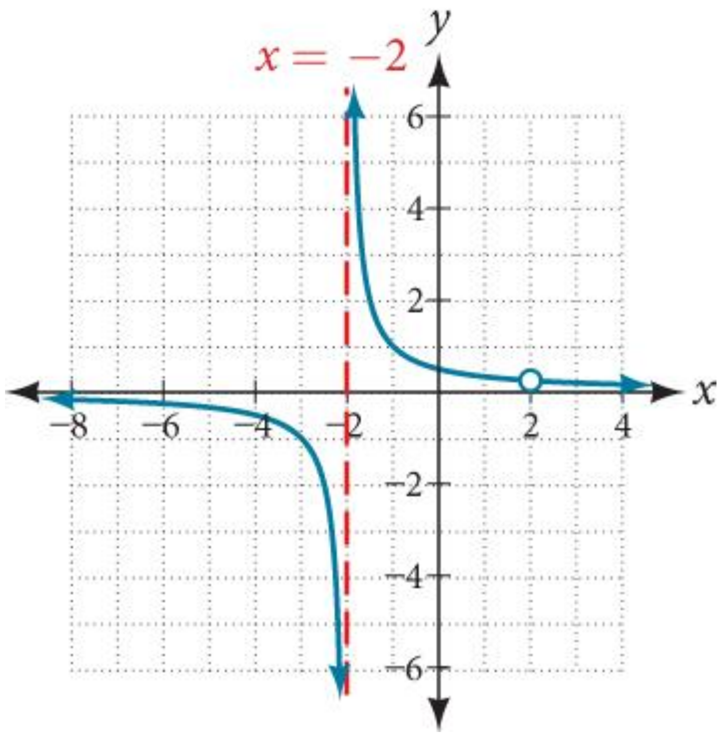
$$f(x) = \frac{x-2}{x^2 - 4}$$

Factor the numerator and the denominator.

$$k(x) = \frac{x-2}{(x-2)(x+2)}$$

Notice that there is a common factor in the numerator and the denominator, $x-2$. The zero for this factor is $x = 2$. This is the location of the removable discontinuity.

Notice that there is a factor in the denominator that is not in the numerator, $x + 2$. The zero for this factor is $x = -2$. The **vertical asymptote** is $x = -2$. See the figure.



The graph of this function will have the **vertical asymptote** at $x = -2$, but at $x = 2$ the graph will have a hole.

Practice 2.5-3-1:

Find the horizontal asymptotes of the function.

p

$$f(x) = \frac{x^{3} - 1}{5x^{3} - 7x - 6}$$

q

Based on the rule:

If p>p-1, and q>q-1, a$\neq 0$ and b$\neq 0$

$$f(x) = \frac{ax^p + cx^{p-1}\dots}{bx^q + dx^{q-1}\dots}$$

If the numerator power *p* is less than the denominator power *q*, the horizontal asymptote is *y = 0*

p < q, horizontal asymptote is *y = 0.*

If the numerator power *p* is greater than the denominator power *q*, the horizontal asymptote is *y = 0*

p > q, horizontal asymptote does not exist.

If the numerator power *p* is the same as the denominator power *q*, the horizontal asymptote is *y* $=\frac{a}{b}$

p = q, horizontal asymptote *y* $=\frac{a}{b}$.

Since p = *q, horizontal asymptote* y = $\frac{\mathbf{a}}{\mathbf{b}} = \frac{\mathbf{1}}{\mathbf{5}}$

Practice 2.5-3-2:

Find the horizontal asymptotes of the function.

$$f(x) = \frac{2x^{4} - 1}{5x^{2} + 3x - 6}$$

p

q

Based on the rule:

If p>p-1, and q>q-1, a$\neq 0$ and b$\neq 0$

$$f(x) = \frac{ax^p + cx^{p-1}\dots}{bx^q + dx^{q-1}\dots}$$

If the numerator power p is less than the denominator power q, the horizontal asymptote is $y = 0$

$p < q$, horizontal asymptote is $y = 0$.

If the numerator power p is greater than the denominator power q, the horizontal asymptote is $y = 0$

$p > q$, horizontal asymptote does not exist.

If the numerator power p is the same as the denominator power q, the horizontal asymptote is $y = \frac{a}{b}$

$p = q$, horizontal asymptote $y = \frac{a}{b}$.

Since $p > q$, horizontal asymptote does not exist.