3.5 Answer Key

Practice 3.5-1-1:

Use the Change-of-Base Formula and a calculator to evaluate the logarithm, round it to nearest hundredth.

$$\boldsymbol{log_{\frac{1}{5}}(27)}$$

If use natural log: $\mathbf{log}_{\boldsymbol{b}}(\boldsymbol{x}) = \frac{\mathbf{ln}(\boldsymbol{x})}{\mathbf{ln}(\boldsymbol{b})}$

$$\boldsymbol{log_{\frac{1}{5}}(27)} = \frac{\ln(27)}{\ln\left(\frac{\mathbf{1}}{\mathbf{5}}\right)} = -2.0478185835\ldots \approx \mathbf{-2.05}$$

Answer: $\mathbf{-2.05}$

Practice 3.5-2-1:

Using the product, quotient, and power rules for logarithms, expand the logarithmic expressions. To check if it's fully expanded: 1) No fractions, 2) No multiplication, 3) No exponents.

$$\log_b\left(\frac{xy^{\,8}}{7z^3}\right)$$

1) quotient (change to "-")

$$\log_b\left(\frac{xy^{\,8}}{7z^3}\right) = \log_b(xy^8) - \log_b(7z^3)$$

2) product (change to "+")

$$\log_b(xy^8) - \log_b(7z^3) = (\log_b(\mathrm{x}) + \log_b(y^8)) - (\log_b(7) + \log_b(z^3))$$

3) power (bring the power down)

$$(\log_b(\mathrm{x}) + \log_b(y^{\mathbf{8}})) - (\log_b(7) + \log_b(z^{\mathbf{3}}))$$
$$= (\log_b(\mathrm{x}) + \mathbf{8}\log_b(\mathrm{y})) - (\log_b(7) + \mathbf{3}\log_b(z))$$

Last step: simplify

$$(\log_b(x) + 8\log_b(y)) - (\log_b(7) + 3\log_b(z)) \quad \textbf{distribute the negative sign}$$
$$= \mathbf{\log_b(x) + 8\log_b(y) - \log_b(7) - 3\log_b(z)}$$

Answer: $\mathbf{\log_b(x) + 8\log_b(y) - \log_b(7) - 3\log_b(z)}$

Practice 3.5-3-1:

Using the product, quotient, and power rules for logarithms, condense the logarithmic expressions.

$$2\log_b(x) + \log_b(y) - 3\log_b(z) - \log_b(5)$$

1) power (coefficient)

$$\mathbf{2}\log_b(x) + \log_b(y) - \mathbf{3}\log_b(z) - \log_b(5)$$
$$\log_b(x^{\mathbf{2}}) + \log_b(y) - \log_b(z^{\mathbf{3}}) - \log_b(5)$$

2) product ("+")

$$\log_b(x^2y) - \log_b(z^3) - \log_b(5)$$

3) quotient ("-")

$\log_b(x^2y) \mathbf{- \log_b(z^3) - \log_b(5)}$ All negative logarithmic terms are in the denominator

$= \log_b\left(\frac{x^2y}{z^3 5}\right)$

$= \mathbf{\log_b\left(\frac{x^2y}{5z^3}\right)}$

In math, we prefer to have the coefficient in front, so we reorganized the denominator.

Answer: $\mathbf{\log_b\left(\frac{x^2y}{5z^3}\right)}$