3.6 Answer Key

Practice 3.6-1-1:

Solve the exponential equation.

$$5^x = 2^{x+1}$$

1. Ensure the equation has exponential expressions on both sides with a coefficient of 1, or one side is an exponential expression with a coefficient of 1 and the other side is a constant.

   Yes, so go to step 2.

2. Apply a logarithm (either ln or log) to both sides of the equation.

$$\ln(5^x) = \ln(2^{x+1})$$

3. Use logarithmic properties to bring the exponent down in front.

   Use: $\log_a M^r = r\log_a M$

$$(\mathrm{x})\ln(5) = (x+1)\ln(2)$$

4. Solve for x.

$$\mathrm{x}\cdot\ln(3) = \mathrm{x}\cdot\ln(2) + \ln(2)$$
$$\mathbf{x}\cdot\ln(3) - \mathbf{x}\cdot\ln(2) = \ln(2)$$
$$\mathbf{x}\cdot(\ln(3) - \ln(2)) = \ln(2)$$

Take the common factor x out.

$$\frac{\mathrm{x}\cdot(\ln(3)-\ln(2))}{(\ln(3)-\ln(2))} = \frac{\ln(2)}{(\ln(3)-\ln(2))}$$

$$\frac{\mathrm{x}\cdot\cancel{(\ln(3)-\ln(2))}}{\cancel{(\ln(3)-\ln(2))}} = \frac{\ln(2)}{(\ln(3)-\ln(2))}$$

$$\boldsymbol{x} = \frac{\ln(2)}{\ln(3)-\ln(2)}$$

Hard to understand? Let's try this:

$$\mathrm{x}\cdot 3 - \mathrm{x}\cdot 2 = 3x - 2x$$

So whoe we get the answer? We use 3-2=1 and keep the x. so that

$$3x - 2x = (3-2)x = x$$

Now,what if

$$\mathrm{x}\cdot\ln(3) - \mathrm{x}\cdot\ln(2)$$

We would have

$$(\ln(3) - \ln(2))x$$

Practice 3.6-2-1:

Solve the logarithm equation

$$ln(x) - ln\,(2-x) = ln(3)$$

Step 1: Simplify Each Side: Make sure each side contains only one logarithmic expression each side.

there are **multiple logarithms** on one side, **condense** them:

$$ln(x) \boxed{-} ln\,(2-x) = ln(3)$$

We see "–", thus use the **quotient rule**: $\log_a M - \log_a N = \log_a \left(\frac{M}{N}\right)$

$$\ln\left(\frac{x}{2-x}\right) = \ln\,(3)$$

Step 2: Drop the Logs (If Bases Match)

- After condensing, if both sides have **the same log base**, you can drop the logarithms and set the remaining expressions equal:

$$\ln\left(\frac{x}{2-x}\right) = \ln\,(3)$$

$$\cancel{\ln}\left(\frac{x}{2-x}\right) = \cancel{\ln}\,(3)$$

$$\frac{x}{2-x} = 3$$

Step 3: Solve for the Variable

- Solve the resulting algebraic equation for the unknown variable (e.g., x).

$\frac{x}{2-x} = 3$ To cancel the denominator, we need to multiply both sides by the denominator.

$$(2-x)\frac{x}{2-x} = 3(2-\mathrm{x})$$

$x = 3(2 - x)$

$x = 6 - 3x)$

$x + 3x = 6$

$4x = 6$

$x = \frac{6}{4}$

$x = \frac{3}{2}$

Step 4: Check Solutions

$$ln(x) - ln\,(2 - x) = ln(3)$$

Based on the domain, we must ensure $x > 0$, and $2 - x > 0$, thus we can use a table to check.

Potential Answer	x	$2 - x$
$\frac{3}{2}$	$\frac{3}{2}$ Positive value.✓	$2 - \frac{3}{2} = \frac{1}{2} = 0.5$ Positive value. ✓

Thus $x = \frac{3}{2}$ is the answer.

Practice 3.6-2-2:

Solve the logarithm equation

$$\log_4(x) + \log_4(x-3) = 1$$

Step 1: Simplify Logs: Make sure log side contains only one logarithmic expression.

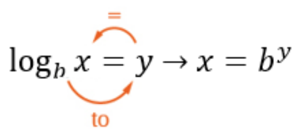
If a logarithm has a coefficient**,**

use the product rule**:** $\log_a M + \log_a N = \log_a(MN)$

$$\log_4((x)(x-3)) = 1$$

Step 2: Arrow method.

- After condensing use arrow method convert it to exponential equation.



$$\log_4((x)(x-3)) = 1$$

$$4^1 = ((x)(x-3))$$

Step 3: Solve for the Variable

- Solve the resulting algebraic equation for the unknown variable (e.g., x).

$$4^1 = ((x)(x-3))$$

$$4 = x^2 - 3x$$

$$0 = x^2 - 3x - 4$$

$$0 = (x+1)(x-4)$$

$$x = -1, x = 4$$

Step 4: Check for Extraneous Solutions

$$\log_4(x) + \log_4(x-3) = 1$$

Based on the domain, we must ensure $x > 0$, $x - 3 > 0$thus we can use a table to check.

Potential Answer	x	$x - 3$
-1	$\mathbf{-1}$ **Negative** value conflicts with the domain, so we exclude -1 as a solution.	
4	$\mathbf{4}$ **Positive** value.✓	$4 - 3 = \mathbf{1}$ **Positive** value.✓

Thus $x = 4$ is the answer.